Topic: Second derivative test

Question: Use the Second Derivative Test to classify the given critical points.

$$f''(x) = -6x + 6,\ x = 0,\ x = 2$$

Answer choices:

A Relative minimum at $x = 0$; Relative maximum at $x = 2$

B Relative minimum at $x = 2$; Relative maximum at $x = 0$

C Relative minima at $x = 0$ and $x = 2$

D Relative maxima at $x = 0$ and $x = 2$

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Solution: A

Since the second derivative of the function is positive at the critical number $x = 0$ ($f''(0) = -6(0) + 6 = 6 > 0$), the function f is concave up at that point. Therefore, there is a relative minimum at $x = 0$.

Since the second derivative of the function is negative at the critical number $x = 2$ ($f''(2) = -6(2) + 6 = -6 < 0$), the function f is concave down at that point. Therefore, there is a relative maximum at $x = 2$.

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Topic: Second derivative test

Question: What are the extrema of the function using the second derivative test?

$$f(x) = x^2 + x + 4$$

Answer choices:

A There is a local minimum at $x = -1/2$.

B There is a local maximum at $x = -1/2$.

C There is a local minimum at $x = 1/2$.

D There is a local maximum at $x = 1/2$.

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Solution: A

Start by finding the critical numbers, which we'll do by taking the first derivative of the function.

$$f(x) = x^2 + x + 4$$

$$f'(x) = 2x + 1$$

Now set the derivative equal to 0 and solve for x.

$$0 = 2x + 1$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

There is one critical number at $x = -1/2$. Now take the second derivative of the function.

$$f''(x) = 2$$

Substitute the critical number into the second derivative.

$$f''\left(-\frac{1}{2}\right) = 2$$

Because the second derivative is positive at the critical number, it means the critical number $x = -1/2$ represents a minimum. Remember when $f''(x) > 0$, there is a local minimum at that point.

The function $f(x) = x^2 + x + 4$ has a local minimum at $x = -1/2$.

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Topic: Second derivative test

Question: What are the extrema of the function using the second derivative test?

$$f(x) = \frac{1}{2}x^4 - x^2 + 6$$

Answer choices:

A There are local maximums at $x = -2$ and $x = 2$ and a local minimum at $x = 0$.

B There are local minimums at $x = -2$ and $x = 2$ and a local maximum at $x = 0$.

C There are local maximums at $x = -1$ and $x = 1$ and a local minimum at $x = 0$.

D There are local minimums at $x = -1$ and $x = 1$ and a local maximum at $x = 0$.

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Solution: D

Start by finding the critical numbers, which we'll do by taking the first derivative of the function.

$$f(x) = \frac{1}{2}x^4 - x^2 + 6$$

$$f'(x) = 2x^3 - 2x$$

$$f'(x) = 2x\left(x^2 - 1\right)$$

$$f'(x) = 2x(x - 1)(x + 1)$$

Now set the derivative equal to 0 and solve for x.

$$0 = 2x(x - 1)(x + 1)$$

$$x = -1,\ 0,\ 1$$

There are three critical numbers at $x = -1,\ 0,\ 1$. Now take the second derivative of the function.

$$f''(x) = 6x^2 - 2$$

Substitute the critical numbers into the second derivative.

$$f''(-1) = 6(-1)^2 - 2 = 4$$

$$f''(0) = 6(0)^2 - 2 = -2$$

$$f''(1) = 6(1)^2 - 2 = 4$$

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Remember when $f''(x) > 0$, there is a local minimum at that point, and when $f''(x) < 0$, there is a local maximum at that point. Therefore, the function has local minimums at $x = -1$ and $x = 1$, and a local maximum at $x = 0$.

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